ACCURACY IMPROVEMENT OF TRAVEL TIME ESTIMATION IN URBAN ENVIRONMENT USING STATE TRANASITION-DEPENDENT TIME-OCCUPANCY

Patcharee PAISITTANAKORN
Department of Electrical Engineering
Chulalongkorn University
Phayathai, Pathumwan, Bangkok, 10330, Thailand
E-mail: jar_psisit@hotmail.com

Chaiyachet SAIVICHIT
Department of Electrical Engineering
Chulalongkorn University
Phayathai, Pathumwan, Bangkok, 10330, Thailand

Wasan PATTARA-ATIKOM
National Electronics and Computer Technology Center (NECTEC)
Phahonyothin, Klong Luang, Pathumthani, 12120, Thailand
E-mail: wasan@nectec.or.th

Abstract: A methodology to estimate travel time with acceptable accuracy in urban environment is proposed. In this study, the traffic states which were used in modeling were classified into three states; free flow state, saturated flow state and over-saturated flow state. These states were defined based on time-occupancy value. The most congested traffic is called over-saturated flow state whereas the least congested traffic is free flow state. However, this travel time estimation model based on time-occupancy alone is not sufficient to estimate accurately by applying only static linear regression method. This is because static linear regression method is a linearization technique. This method cannot capture time-occupancy value during transition period. This paper aims to improve the accuracy of travel time estimation in urban environment using state transition-dependent time-occupancy. We proposed separate models to capture time-occupancy during different transition period and consequently to estimate travel time more accurately. We then applied the concept of dynamic time-series regression (ARIMAX). The results revealed that our proposed separate models can improve the accuracy of travel time estimation based on only time-occupancy.

Key Words: travel time estimation, time occupancy, state transition dependence, urban road

1. INTRODUCTION

Recently, Traffic Information System has been increasingly developed in order to report useful traffic information to the motorists on a highway or even on an urban roadway. Apart from those advantages, the system seems to be more complex if implemented on the urban road way. One of the systems used in Bangkok nowadays is the Traffic Sign Board which reports the Congestion Color Levels, i.e. Green, Amber and Red. Although, reporting Congestion Color Levels could be useful to the motorists, it is only qualitative traffic information. Therefore, reporting quantitative traffic information, such as, Travel Time becomes the motivation and challenges for authors. Many researchers have proposed the schemes of Travel Time Prediction, i.e. J.W.C. van Lint *et al.* (2006), Angshuman, (2006), Daniel Billing *et al.* (2006), Zhi-Peng *et* *al*. (2008), and John Rich *et al*. (2004). From their schemes, the reliability of their models depends on the amount of collected data which means that they need to use a large amount of data to guarantee the reliability of the model. As a result, the cost will be gained as they increase the number of collected data. In Bangkok, reporting travel time information on urban roadway is not suitable to happen yet. This is because current traffic equipments, such as, traffic camera are still incapable of collecting enormous traffic data to support travel time information. If we upgrade these equipments, it would need a large investment. Traffic equipments have been installed on roadside around Bangkok. They have installed only one or two camera(s) per a road segment (A road segment represents distance between a signal-controlled intersection upstream to another signalcontrolled intersection downstream) for detecting vehicles and applying Image Processing to generate time-occupancy value as in reference of Markos Papageorgiou *et al.* (2008). From the inadequacy problem stated above, we are interested on estimating travel time based on only available traffic data from current situation which is time-occupancy. Benefits of our proposed concept are to save cost to service provider for equipment installation and also to be able to report more useful traffic data such as travel time information on road segments instead of Congestion Color Levels to motorists. In the proposed methodology to estimate travel time based on only time-occupancy in urban environment, traffic states which were used in modeling were classified into three states as shown in Fig.1. The first state is "*free flow*" state which means that vehicles will not get disturbed by the others. They can accelerate their speed up to a maximum speed. The second state is "*saturated*" flow state. Flow value is stable whereas the density value is increasing until it reaches some density values. The third state is "*over-saturated*" flow state. It represents traffic flow that is influenced by the effects of downstream bottleneck.

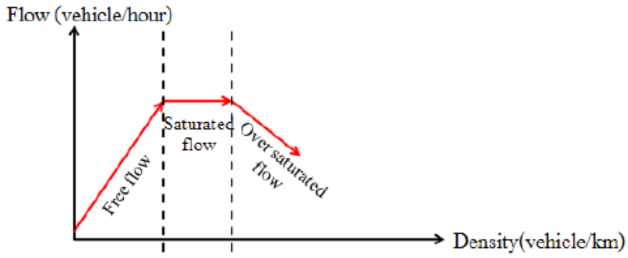

Figure 1. Fundamental diagram of traffic flow in theory

Travel time estimation models based on time-occupancy using data from traffic simulator to analyze travel time and time-occupancy relationships have already been proposed by the previous works. Sisiopiku *et al.* (1994) proposed travel time estimation model by applying "Single Linear Regression" model with piece-wise linear method. For the work of Gault (1982) and Gipps (1976), they applied "Multiple Linear Regression" model in their estimation methods. In this case, the disadvantage is that it requires traffic data that cannot routinely collect from detectors such as the arrival time of a vehicle at a detector. However, the previous works,i.e. travel time estimation based on time-occupancy using only static regression model performs reasonably well in free flow state. Nevertheless, they do not perform as well in situation of dependency on past knowledge dependence such as saturated flow state and over-saturated flow state. This is because the residuals information from past effects on current information. Therefore, this paper focuses on accuracy improvement of travel time estimation based on state transition-dependent time-occupancy in urban environment especially in state transition period from saturated flow state into over-saturated flow state. We selected Autoregressive Integrated Moving-average with External input series (ARIMAX) to track the collected data. We observed and collected travel time and time-occupancy data from microscopic traffic simulator (MITSIMLab) "MITSIMLab is a simulation-based laboratory that was developed for evaluating the impacts of alternative traffic management system designs at the operational level. The various components of MITSIMLab are organized in three modules: microscopic traffic simulator (MITSIM), traffic management simulator (TMS) and graphical user interface (GUI). MITSIMLab is adapted to model traffic flow in the simulation. This level of detail; with microscopic approach, is necessary for an evaluation at the operational level. The traffic management simulator (TMS) represents the candidate traffic control and routing logic under evaluation. The control and routing strategies generated by the traffic management module determine the status of the traffic control and route guidance devices. Motorists will respond to the various traffic controls and guidance while interacting with each other, Yang *et al.* (2000) and Moshe Ben-Akiva et al. (2001)" The remainder of this paper is organized as followed: Section 2 is the static linear regression analysis. Dynamic time series regression is described in section 3. Proposed models are described in section 4. Results are shown in section 5 and 6. And conclusion is in section 7.

2. STATIC LINEAR REGRESSION ANALYSIS

2.1 Experiment Design for traffic simulator on static linear regression analysis

The road segment has been designed for this simulation experiment. We set a signalcontrolled intersection at downstream (the stop line) and the road segment covers a total length of 1500 meters with 2-lanes, placing 1-detector in the middle of the road as in Figure 2. We first calibrated appropriate traffic data for the study of correlation between link travel time and time-occupancy. The appropriate traffic data was chosen for a consideration, i.e. the entry link flows. The entry link flows as system input was used to measure Time-Occupancy value, system output, and was calibrated until it got the whole range of time-Occupancy varied from 0 to 100 percent.

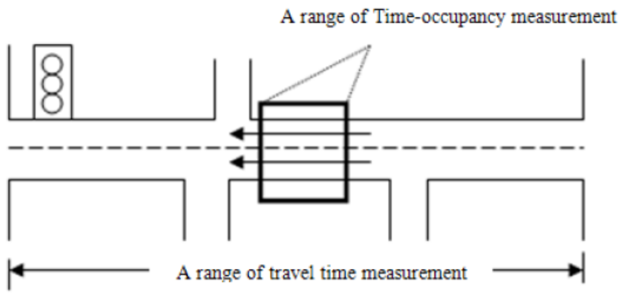

Figure 2. Observation of travel time and time-occupancy data in simulation

2.2 Classification of Traffic States based on time-occupancy value for static linear regression analysis

This part discusses on the correlation between travel time and time-occupancy value which was generated by varying entry link flows. By varying the flow value for each observation time period, traffic simulator generates 120 data points of averaged travel time and timeoccupancy per two hours (i.e. observation time period) which means that each data point in this graph was observed at every one minute as illustrated in Figure.3

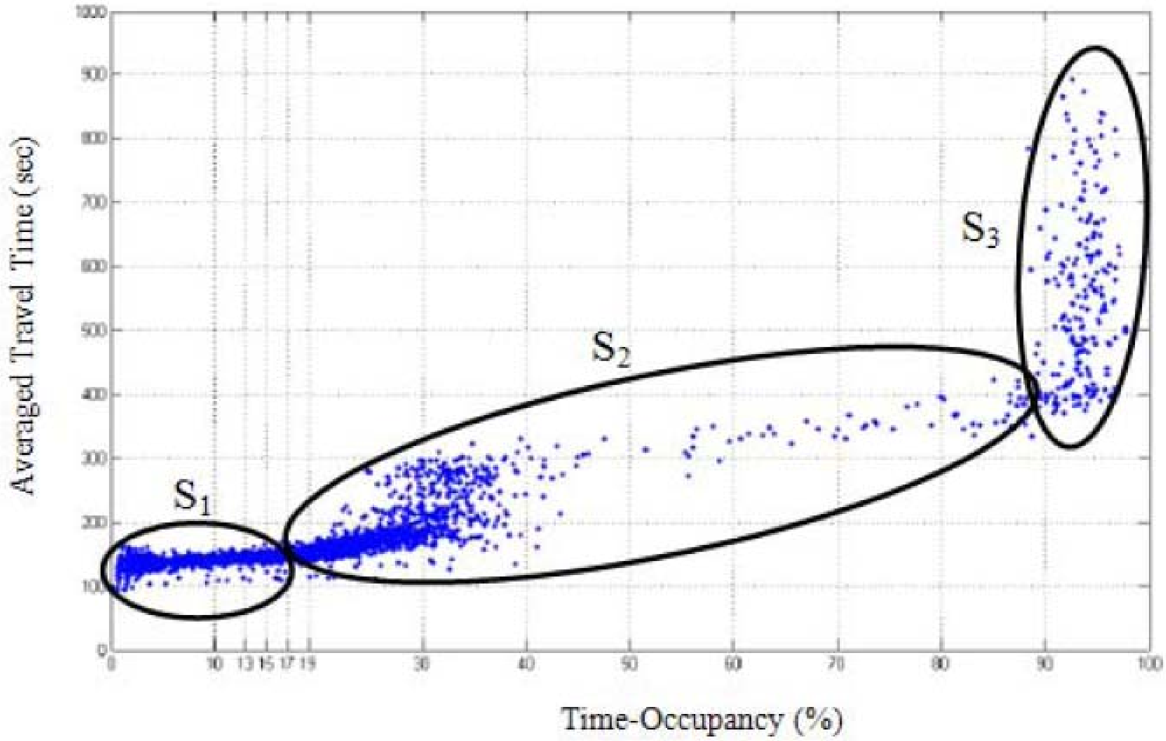

Figure 3. Averaged travel time vs. time-occupancy plot

In Figure.3, the graph could be clearly divided into 3 parts and each part is defined as each traffic state based on time-occupancy value which could be concluded in Table 1.

Table 1. Traffic state classification for static linear regression analysis

Traffic State	Time-occupancy (%)	Symbol
Freeflow (ff)	$0 \leq O < 17$	S^{ff}
Saturate-flow (sf)	$17 \leq O < 90$	S^{sf}
Over-sat flow (of)	$O \geq 90$	S^{of}

2.3 Static regression model with piece-wise linear method

Sisiopiku *et al.* (1994) applied this method to estimate travel time using time-occupancy

$$T=a+b_1\times O+b_2\times\delta\times(O-17)+b_3\times\kappa\times(O-90) \quad (1)$$

$$T=a+b_1\times O+b_2\times KNOT1+b_3\times KNOT2 \quad (2)$$

Where
T: the travel time (T in second) ;
O : the percent time-occupancy;
a, b_1, b_2, b_3: regression parameters;
$KNOT1 = \delta \times (O-17)$;
$KNOT2 = b_3 \times K \times (O-90)$;

$$\kappa = \begin{cases} 1 & \text{if } O > 90; \text{ and} \\ 0 & \text{otherwise} \end{cases}$$

$$\delta = \begin{cases} 1 & \text{if } O > 17; \text{ and} \\ 0 & \text{otherwise} \end{cases}$$

“Note that the division points of time-occupancy in this static linear regression model are dependent on traffic condition and specific site, Sisiopiku *et al.* (1994)”

2.4 Results of applying Static linear regression model

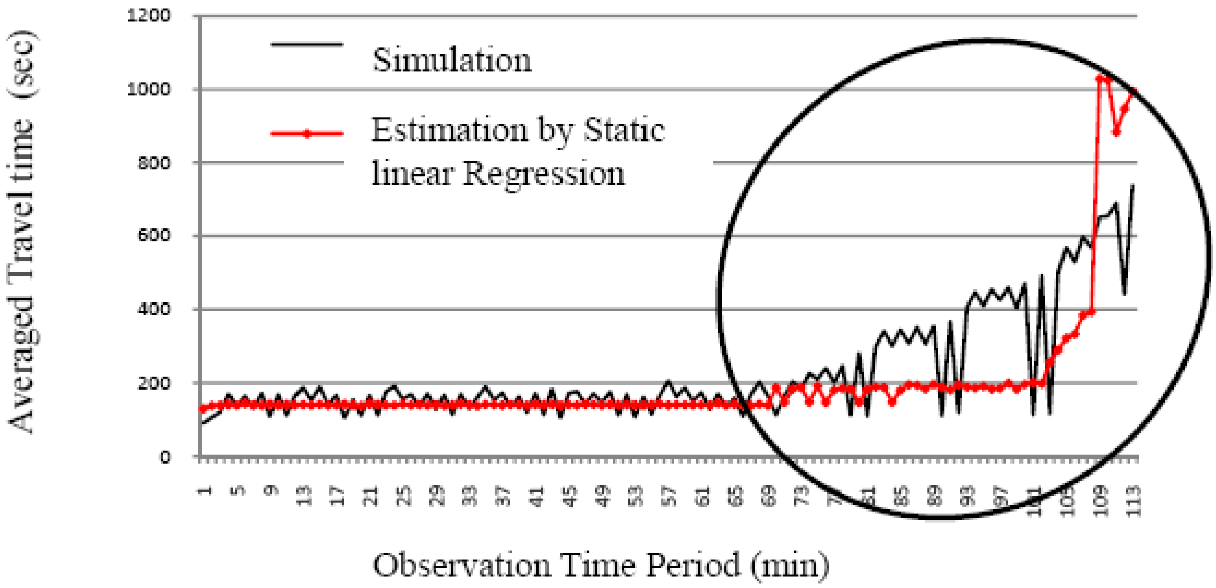

Figure 4. Result comparison between estimated travel time by static linear regression method and simulation

Time-occupancy value has been divided into three states as shown in Table 1 and they are independent of each other. This implies that the transition between states is unquestionably ignored. This paper proposed the separate state transition patterns by applying dynamic time series regression, i.e. Autoregressive Integrated Moving-average with External input series (ARIMAX), to improve the accuracy of travel time estimation especially the circle area as in Figure. 4.

3. DYNAMIC TIME SERIES REGRESSION

3.1 ARIMAX approach

As in references of Box *et al.* (1970), Daniel Billings *et al.* (2006) and Jing Fan *et al.* (2009), "ARIMAX approach allows us to combine linear regression and ARIMA process into one model. It broadens the applicability of ordinary least squares modeling. ARIMAX can be divided to 4 parts as followed:

- Autoregressive (AR) of order p is used for improving the current travel time estimation using previous value of travel time and a random noise.

$$T_t = \Phi_1 T_{t-1} + \Phi_2 T_{t-2} + \ldots + \Phi_p T_{t-p} + Z_t \quad (3)$$

Where $\Phi = \{\Phi_1, \Phi_2 \ldots \Phi_p\}$ are autoregressive coefficients.
Z_t: the disturbance at time t. The process $\{Z_t\}$ modeled as an independent and identically distributed (*iid*) white noise with zero mean and variance $\sigma 2$.That is, $E[Z_t] = 0$ and $E[Z_t^2] = \sigma^2$ for all t and $E[Z_s Z_t] = 0$ if $t \neq s$

- Moving-average (MA) of order q is used for improving the current travel time estimation using previous value of travel time estimation error and then

$$T_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} + \ldots + \theta_q Z_{t-q} \quad (4)$$

- Where $\theta = \{\theta_1, \theta_2 \ldots \theta_q\}$ are moving average coefficients. Normally, the term of ARMA (p,q) is always written in the back shift operator *'B'* where the back shift operator *'B'* operates on an element of a time-series to produce the previous element as $B^i X_t = X_{t-1}$, so ARMA(p,q) can be written as

$$(1-\Phi_1 B-\Phi_2 B^2-\ldots-\Phi_p B^p)\, T_t = (1-\theta_1 B-\theta_2 B^2-\ldots-\theta_q B^q)\, Z_t \qquad (5)$$

- Differencing (*I*) of order *d* is used for converting non-stationary travel time series data and time-occupancy series data to stationary data written in term of the back shift operator *'B '* i.e. $(1-B)^d T_t$, where *d* is the number of differencing order, *B* is the back shift operator, and T_t denotes the current travel time. For example $(1-B)^1 T_t = T_t - T_{t-1}$etc., Daniel Billings *et al.* (2006) "
- *r* in ARIMAX application is represented by the external input time series which is time-occupancy. It can be written in term of its previous and current value in order '*r*'.
- Finally, ARIMAX (*p,d,q,r*) can be written as followed,

$$T_t = \mu + \sum_{n=0}^{r} \beta_n O_{t-n} + \frac{(1+\theta_1 B+\theta_2 B^2+\ldots+\theta_q B^q)}{(1-B)^d (1-\phi_1 B-\phi_2 B^2-\ldots-\phi_p B^p)} Z_t \qquad (6)$$

where
T_t : travel time at the current time.
O_{t-n} : time-occupancy, for $n = 0$ to r.
μ : a constant.
d : the number of differencing of travel time and time-occupancy.
β_n: input coefficient at 'n^{th}', for $n = 0$ to r.
ρ_i : coefficient of previous value of travel time, for $i = 1$ to p.
Z_{t-j} : coefficient of previous value of travel time forecast error, for $j = 1$ to q.
Z_t : white noise with zero mean.

3.2 ARIMAX Steps

ARIMAX model can be divided into three steps. The first step is "Model Identification". The objective of this step is to define p, d, q and r which are the order of autoregressive, differencing, moving average and external input, respectively. The second step is "Model Estimation". The objective of this step is to estimate model parameters by using ordinary least square. The third step is "Diagnostic Checking". The objective of this step is to check the fitted models by "Serial Correlation" test. Then, we select the most fitted model by considering several statistical measurements such as Adjusted R2, SEE, SC, AIC, BIC, etc. For example, a good model should have small SEE, SC and AIC value and big Adjusted R_2 value.

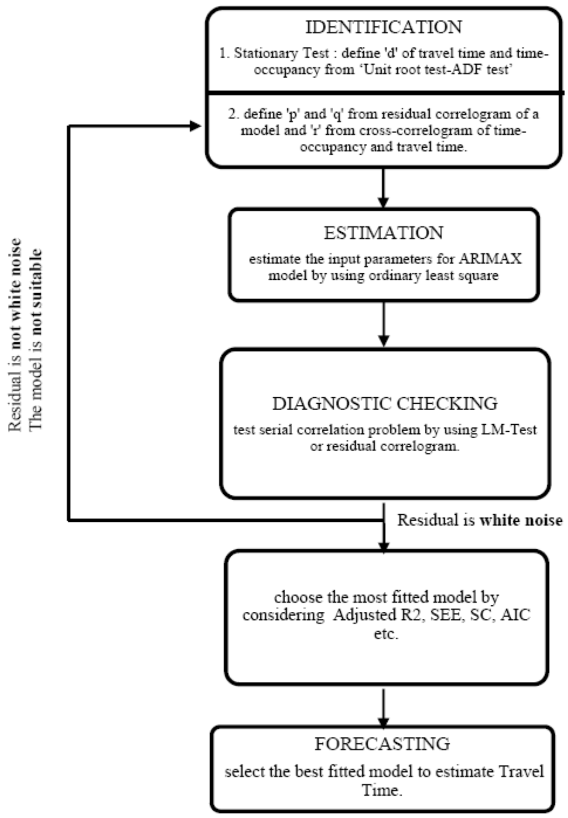

Figure 5. Analysis steps of ARIMA (p,d,q) with intput data series(r)

ARIMAX mechanism models the residual structure. Its concept is to minimise the residual of a model by combining the previous values of output (AR(p)), the previous values of the forecast error (MA(q)), the previous/current values of external input series (i.e. timeoccupancy) and the differencing (I(d)) for non-stationary time series data.

4. PROPOSED MODELS

4.1 Definition of State Transition Patterns

This paper focuses on the accuracy improvement of travel time estimation based on the change of time-occupancy, i.e. state transition dependent, especially from saturated flow state to over-saturated flow state as shown in Figure. 6.

From this figure, the transition has much significant effect to the accuracy of travel time estimation. However, the change from saturated flow state to over-saturated flow state also relates to free flow state in sequential form, namely free flow, saturated flow and over-saturated flow. Therefore, we proposed separate models to cover the four state transition patterns as followed:

- $\mathbf{S^{ff}_{t-1} \rightarrow S^{ff}_{t}}$

No change of state as the time-occupancy range of O_{t-1} and O_t are both within range of free flow state.

- $\mathbf{S^{ff}_{t-1} \rightarrow S^{sf}_{t}}$

The time-occupancy value within the range of free flow state at time (t-1) (O_{t-1}) changes to the saturated flow state at time t (O_t) i.e. $O_{t-1} < O_t$.

- $\mathbf{S^{sf}_{t-1} \rightarrow S^{sf}_{t}}$

No change of state as the time-occupancy range of O_{t-1} and O_t are both within range of saturated flow state.

- $\mathbf{S^{sf}_{t-1} \rightarrow S^{of}_{t}}$

The time-occupancy value within the range of saturated flow state at time (t-1) (O_{t-1}) changes to the over-saturated flow state at time t (O_t) i.e. $O_{t-1} < O_t$. From these four proposed models, we have applied Dynamic Time Series Regression (ARIMAX) to track travel time value which occurs during state transition. Details of state classification with time-occupancy range are shown in Table.2

Table 2. Proposed state classification for dynamic time series regression analysis

Previous State	Present State	Previous O_{t-1} range (%)	Present O_t range (%)	Symbol
Free flow(ff)	Free flow(ff)	$0 \leq O_{t-1} < 17$	$0 \leq O_t < 17$	$S^{ff}_{t-1} \rightarrow S^{ff}_t$
Free flow (ff)	Saturate-flow (sf)	$0 \leq O_{t-1} < 17$	$17 \leq O_t < 35$	$S^{ff}_{t-1} \rightarrow S^{sf}_t$
Saturate-flow (sf)	Saturate-flow (sf)	$17 \leq O_{t-1} < 35$	$17 \leq O_t < 35$	$S^{sf}_{t-1} \rightarrow S^{sf}_t$
Saturate-flow (sf)	Over-sat flow (of)	$35 \leq O_{t-1} < 90$	$35 \leq O_{t-1} < 90$	$S^{sf}_{t-1} \rightarrow S^{of}_t$

4.2 Data preparation for training sets

This part explains the data preparation plan for training sets in ARIMAX model. Time occupancy and travel time data were used as the training data sets. We then designed the separation of training sets which are suitable to each proposed state pattern. From the experimental results of varying flow as the deterministic system, it reveals that the observation time period has a significant effect on travel time especially in saturated flow state illustrated in Fig.6. Therefore, we have to define the observation time period optimally for each proposed state pattern due to the fact that the observation time period that affects travel time will be different from other proposed state patterns. Note that we defined the observation time period just only in the process of separating the training sets. Because the traffic states, i.e. the free flow, saturated flow and over-saturated flow sates occurred in sequential stage, we have to classify the data to the right model proposed in section 4.1 in order to calculate the parameter in ARIMAX model by using that already categorized data. The separation approach is categorized as followed (Figure. 6),

For free flow state, we use the data within the time range of 0th to 30th minute, i.e. before the state of changing to saturated flow state, namely Transition#1.

1. For Transition#1, the data between 30th to 33rd minute (the state of changing to saturated flow) is being chosen.
2. For saturated flow state, we picked the data that is within the range from 33rd to 53rd minute before the state of changing to over-saturated flow state, namely Transition#2.
3. For Transition#2, we selected the data from 53rd to 61st minute (the state of changing to over-saturated flow).

From this design, it can be concluded into an appropriate observation time period of data separation for each state pattern illustrated in Table 3.

Table 3. Time period of each state for the training sets

States	Time Period (minute)
$S^{ff} \rightarrow S^{ff}$ (free flow)	30
$S^{sf} \rightarrow S^{sf}$ (saturated flow)	20
$S^{ff}_{t-1} \rightarrow S^{sf}_{t}$ (Transition #1)	3
$S^{sf}_{t-1} \rightarrow S^{of}_{t}$ (Transition #2)	8

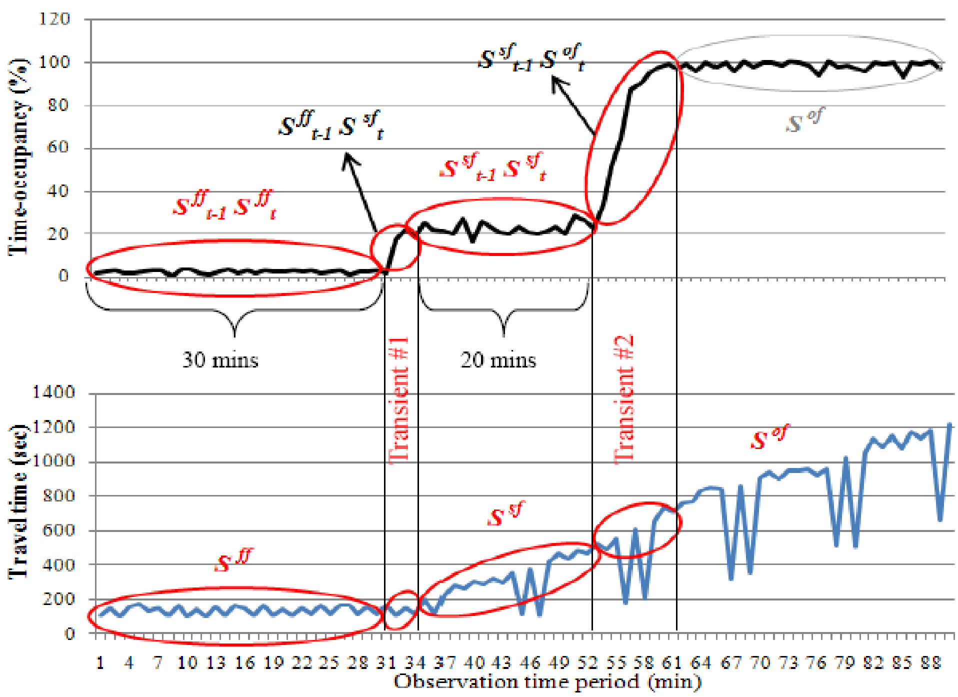

Figure 6. An example of the training data sets separation

4.3 Fitted Models for Proposed State Patterns

By applying Eq.6 with ARIMAX models in each state transition patterns, we finally get four equations represented by Table 4

Table 4. Proposed models applying ARIMAX

State Patterns	Proposed Fitted Models
$S^{ff}_{t-1} \rightarrow S^{ff}_{t}$	$T_t = 100.23+0.75O_t+1.07O_{t-2}+1.13O_{t-4}+ \dfrac{(1-0.87B^{11})}{(1-0.05B-0.99B^{11})} Z_t$
$S^{ff}_{t-1} \rightarrow S^{sf}_{t}$	$T_t = 1.06-4.31O_t -5.39O_{t-2}+3.02O_{t-3} + \dfrac{(1+0.2B-0.8B^5)}{(1-B)(1+0.98B+0.84B^2+0.93B^3+0.82B^4)} Z_t$
$S^{sf}_{t-1} \rightarrow S^{sf}_{t}$	$T_t = 1.02-0.2O_t-1.8O_{t-6}-1.8O_{t-8}+ \dfrac{(1-0.83B^9)}{(1-B)(1-0.51B^9)} Z_t$
$S^{sf}_{t-1} \rightarrow S^{of}_{t}$	$T_t = 304.94+12.4O_t-17.47O_{t-1}+9.36\, O_{t-4}+ \dfrac{(1- 0.98B^8)}{(1+0.69B)} Z_t$

5. TEST DATA SET FOR PROPOSED SEPARATE MODELS

This part focuses on the accuracy test of our proposed models. First, we fed our test data, i.e. time-occupancy into the program of choosing models illustrated in Fig. 8. Then, the program processed and returned the sequence of 1, 1.5, 2 and 2.5 which means that we have to apply $(S^{ff}_{t-1} \rightarrow S^{ff}_{t})$,$(S^{ff}_{t-1} \rightarrow S^{sf}_{t})$,$(S^{sf}_{t-1} \rightarrow S^{sf}_{t})$,and $(S^{sf}_{t-1} \rightarrow S^{of}_{t})$respectively.

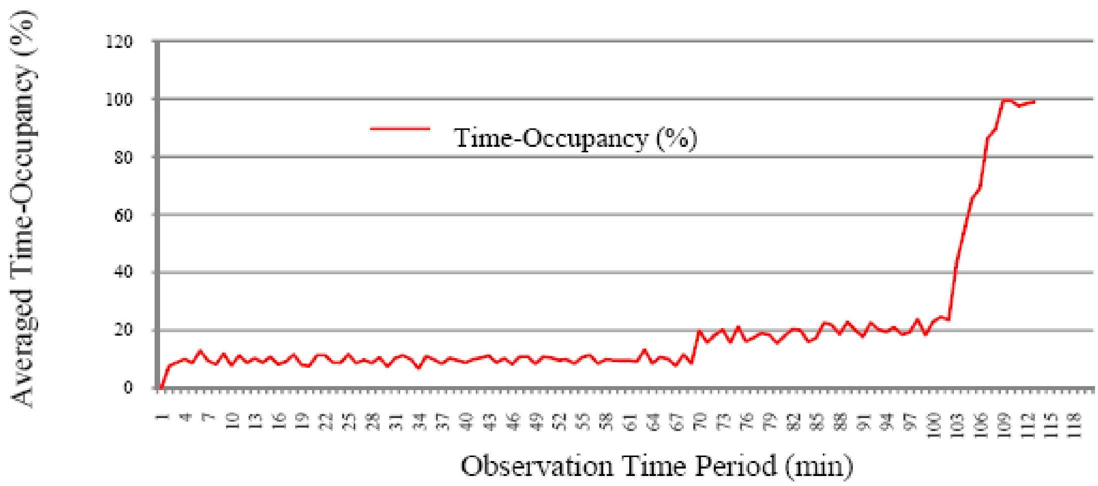

Figure 7. Test data set of the time occupancy data

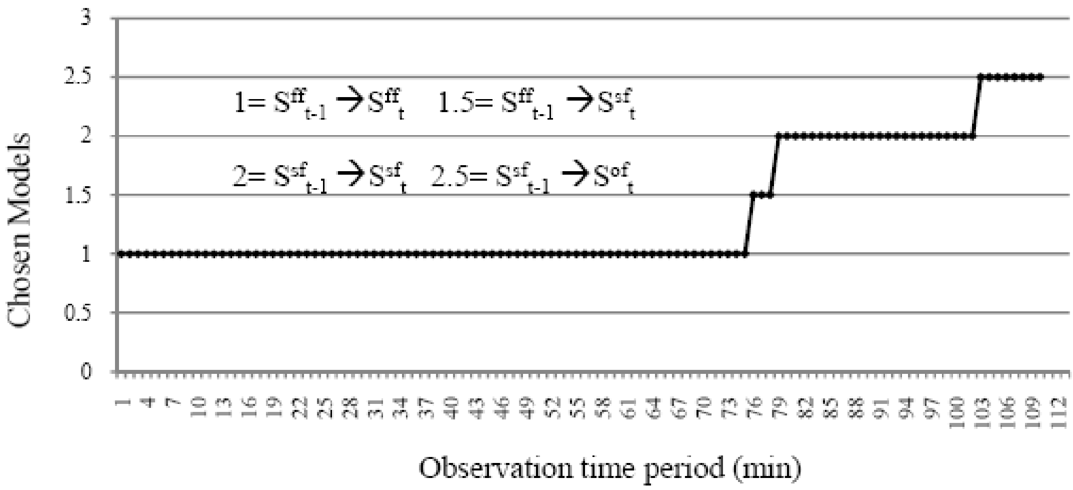

Figure 8. The models which are selected by checking the variance of time occupancy data

6. RESULTS

From the test data set in part 5, the result revealed that travel time estimation in free flow state using "Static Linear Regression Model" can estimate the travel time as accurately as our proposed model ($S^{ff}_{t-1} \rightarrow S^{ff}_{t}$). This is because vehicles will not get disturbed by the others, so they could accelerate their speed up to maximum.

Considering the state transition, saturated flow state to the beginning of oversaturated flow state has some spikes occurred in simulated travel time. These spikes were caused by the traffic light that returned the travel time and time occupancy value when it was turned on. Travel time estimation with static linear regression model cannot estimate accurately when compared with our proposed models ($S^{ff}_{t-1} \rightarrow S^{sf}_{t}$),($S^{sf}_{t-1} \rightarrow S^{sf}_{t}$),and ($S^{sf}_{t-1} \rightarrow S^{of}_{t}$) as shown in Fig. 9. This is because vehicles will be affected by the others especially the past residuals. Finally, we validate our proposed models by using mean absolute percent error as shown in Table. 5. This table shows that our proposed model ($S^{ff}_{t-1} \rightarrow S^{sf}_{t}$),($S^{sf}_{t-1} \rightarrow S^{sf}_{t}$),and ($S^{sf}_{t-1} \rightarrow S^{of}_{t}$)can improve the accuracy of travel time estimation from the previous method by 19.91%, 24.67% and 20.69%, respectively.

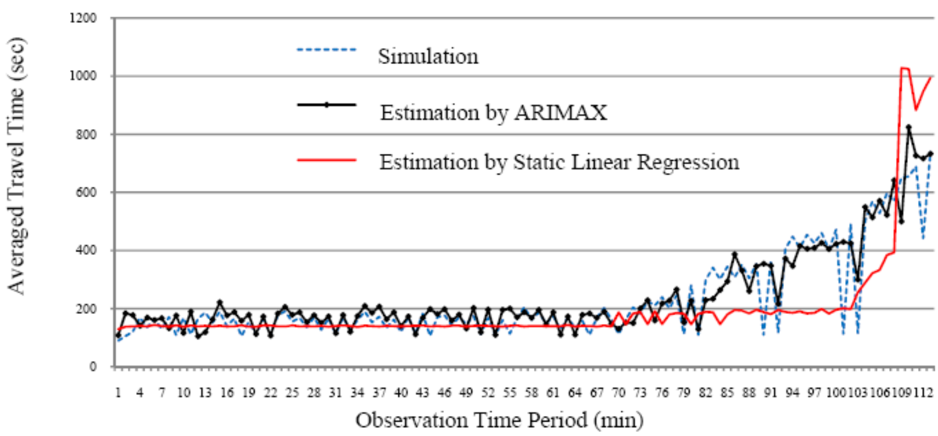

Figure 9. The result of estimated travel time series based on the proposed models

Table 5. Validation of the proposed models by mean absolute percent error (%)

State Patterns / Model types	$S^{ff}_{t-1} S^{ff}_{t}$	$S^{ff}_{t-1} S^{sf}_{t}$	$S^{sf}_{t-1} S^{sf}_{t}$	$S^{sf}_{t-1} S^{of}_{t}$
Static Regression	17.9%	30.83%	51.7%	51.9%
ARIMAX	18.1%	10.92%	27.05%	30.21%

7. CONCLUSION

Travel time estimation based on time-occupancy using only static regression model is not sufficient for the case where past state has influence on the current state such as saturated flow state (S^{sf}) and over-saturated flow state (S^{of}). Transition period cannot be captured by Static Regression Analysis because it occurs during the state change from saturated flow state to over-saturated flow state. Therefore, this paper focuses on accuracy improvement of travel time estimation based on time-occupancy which depends on the state transition. We applied "Dynamic Time-Series Regression" instead of "Static linear Regression" and also proposed four state patterns i.e. ($\mathbf{S^{ff}_{t-1} \rightarrow S^{ff}_{t}}$),($\mathbf{S^{ff}_{t-1} \rightarrow S^{sf}_{t}}$),($\mathbf{S^{sf}_{t-1} \rightarrow S^{sf}_{t}}$), and ($\mathbf{S^{sf}_{t-1} \rightarrow S^{of}_{t}}$). The experiment results show that the vehicles move independently in the free flow state ($\mathbf{S^{ff}_{t-1} \rightarrow S^{ff}_{t}}$); as a result, the estimated travel time of our model and of the previous method (static linear regression) will not have much difference due to the fact that the past residual does not affect current travel time. In saturated flow state ($\mathbf{S^{sf}_{t-1} \rightarrow S^{sf}_{t}}$), the travel time will be increasing, as the number of vehicles left from the past increases. From the experiment, our proposed model for this state can improve the accuracy of travel time estimation because it is the time-dependent estimator. ($\mathbf{S^{ff}_{t-1} \rightarrow S^{sf}_{t}}$) and ($\mathbf{S^{sf}_{t-1} \rightarrow S^{of}_{t}}$) are the transition states which change from the free flow state to the saturated flow state and from the saturated flow state to over-saturated flow state, respectively. In this transition state, the travel time is affected by the residual from the past, therefore, the proposed model will be more efficient compared to the previous method (static linear regression) because the previous method can neither capture nor estimate the travel time in this transition state. However, the travel time estimation is not only affected by Time-occupancy alone, but it is also influenced by another parameter, i.e. Flow. In actual environment, Flow diversely occurs in many levels; as a result, it can have an impact on the accuracy of our estimation. Without considering Flow parameter, we focused on using only the available traffic data, i.e. Time-occupancy, in order to effectively estimate the travel time with more efficient method compared to the previous one. Therefore, the time series analysis was selected here because it is the statistical tool which depends on time. As for the actual implementation, variance of the time-occupancy data must be checked in order to select an appropriate model.

8. ACKNOWLEDGMENT

The authors would like to express a big gratitude for funding support from Thailand Graduate Institute of Science and Technology on this research work and also the author would like to thank you Mr. Patrachart Komolkiti for a valuable time reviewing the manuscript.

REFERENCES

Angshuman Guin, "Travel Time Prediction using a Seasonal Autoregressive Integrated Moving Average Time Series Model," **IEEE Intelligent Transportation System Conference,** September 2006.

Box, George and Jenkins and Gwilym, "Time series analysis: Forecasting and control," San Francisco: Holden-Day, 1970.

Daniel Billing and Jian-Shiou Yang, "Application of the ARIMA Models to Urban Roadway Travel Time Prediction-A case Study," **IEEE International Conference on System, Man, and Cybernetics,** October 2006. Gault, "An on-line Measure of Delay in Road Traffic Computer-Controlled System," **Traffic Engineering and Control, vol. 22, No. 7, pp. 384-389,** 1981.

Gipps, "MULTSIM for Simulating Output from Vehicle Detectors on Multi-lane Signal Controlled Road," **Transport Operation Research Group,** 1976.

J.W.C. van Lint and M. Schreuder, "Travel time prediction for VMS panels—Results and lessons learnt from a scale evaluation study in the Netherlands," **Trans. Res. Board Annu,** Washington DC,2006.

Jing Fan, Rui Shan, Xiaoqin Cao and Peiliang Li, "The Analysis to Tertiary-industry with ARIMAX Model," **Journal of Mathematics Research, vol. 1, No. 2,** September 2009.

John Rich and Erik van Zwet, "A Simple and Effective Method for predicting Travel Time on Freeways," **IEEE Transaction on Intelligent Transportation System, vol. 5 No. 3,** September 2004.

Ling Leng1, Tianyi Zhang1, Lawrence Kleinman 2, Wei Zhu1, "Ordinary Least Square Regression, Orthogonal Regression, Geometric Mean Regression and their Applications in Aerosol Science," **Journal of Physics: Conference Series 78,** 2007.

Markos Papageorgiou and Georgios Vigos, "Relating time-occupancy measurement to spaceoccupancy and link vehicle-count" **Transportation research, part C,** 2008.

Moshe Ben-Akiva, Margaret Cortes, Angus Davol, Haris Koutsopoulos and Tomer Toledo, "MITSIMLab: Enhancements and Applications for Urban Network," 2001.

Sisiopiku, V.P., N.M. Rouphail and A. Santiago. "Analysis of Correlation Between Arterial Travel Time and Detector Data from Simulation and Field Studies," **Transportation Research Record 1457, pp. 166-173.,** 1994.

Sisiopiku, V.P., N.M. Rouphail and A. Santiago. "Travel Time Estimation From Detectors Data for Advanced Traveler Information Systems Applications," Chicago, Illinois University **Transportation Research Consortium,** 1994.

TC Hsia, “System identification: Least-squares methods (Book),” Lexington, Mass., DC Heath and Co., 1977. 177 p, 1977.

Yang, Q., Koutsopoulos and Ben-Akiwa, “A Simulation Laboratory for Evaluating Dynamic Traffic Management Systems,” **Transportation Research Record, 1710, pp. 122-130,** 2000.

Zhi-Peng, YU Hong, LIU Yun-Cai, LIU Fu-Qiang, “An Improved Adaptive Exponential Smoothing Urban Arterial Street,” **Acta Automatica Sinica, vol. 34, No. 11,** November 2008.